Oxford and Cambridge Schools Examination Board.

59/100 C

School Certificate Examination.

10

ELEMENTARY MATHEMATICS I.

Monday, July 11th, 1949. 2 Hours.

[Answer all the questions in Section A (1–4) and not more than **five** *of the questions in Section B (5–12).*

Logarithms, slide-rules, or algebra may be used in any question, unless directions to the contrary are given.]

Section A.

1. An athletic ground is in the shape of a rectangle with a semi-circle on each of the shorter sides. The rectangle is 110 yd. long and 70 yd. wide. Find the distance round the ground, taking π equal to $\frac{22}{7}$.

440

If a boy runs this distance in 65 sec., what is his average speed in miles per hour? Give your answer to the nearest mile.

14

2. Bricks for building a house are piled close together in a stack which is 33 ft. long, 9 ft. wide and 6 ft. high; each brick measures 9 in. by $4\frac{1}{2}$ in. by 3 in. How many bricks must be added to the stack to make the total number of bricks 26,000?

24 998

3. Solve the simultaneous equations

$$3y - 2x = 5, \quad 8y - 5x = 14\frac{1}{6}.$$

$2\frac{1}{2}$ $3\frac{1}{3}$

4. Prove that an exterior angle of a triangle is equal to the sum of the two interior opposite angles. (The angle-sum formula of a triangle, if used, must first be proved.)

In a triangle ABC the side BA is produced to O, and the bisectors of the angles OAC and OBC intersect at P. Prove that the angle APB is half the angle ACB.

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5. Change the subject of the formula

$$t = 2\pi \sqrt{\frac{l}{g}}$$

$l = \dfrac{t^2 g}{4\pi^2}$

so as to express l in terms of t, g and π.

Find the value of l when $t = 0.93$, $g = 981$ and $\pi = 3.142$. Give your answer to three significant figures.   21.5

6. In a parallelogram $ABCD$ the lengths of the sides AB and AD are respectively 5 in. and 3 in., and the length of the diagonal BD is 6 in. Find the length of the diagonal AC to the nearest hundredth of an inch; also calculate the angle ABD to the nearest minute.

7. Sir Isaac Newton constructed a thermometer with a scale on which he marked two points, namely the freezing point of water $0°$ and bloodheat $12°$. The two corresponding points on the Fahrenheit scale are $32°$ and $98°$. Find graphically the point on Newton's scale which corresponds to the point $54°$ on the Fahrenheit scale. Take 1 in. (or 2 cm. if you use centimetre paper) to represent $10°$ on each scale.   4

8. Three numbers a, b and c, of which a is the greatest and c the least, are such that the difference between the first and second is equal to the difference between the second and third. Express this relation by an algebraical equation and find the value of b in terms of a and c.

Also prove that the square of the second number is equal to the number obtained by adding the product of the first and third to the square of the difference between the first and second.

9. A straight line touches a circle, and from the point of contact a chord is drawn. Prove that the angles which the chord makes with the tangent are equal to the angles in the alternate segments.

Two circles intersect at A and B. The tangent at A to the first circle meets the second circle again at P, and the tangent at A to the second circle meets the first circle again at Q. Prove that the line AB (produced if necessary) bisects the angle PBQ.

10. Two straight roads cross each other at right angles at a point O. On one road a man, 16 miles from O, starts to walk towards O at 4 miles an hour. At the same moment another man, who is on the other road 12 miles from O, starts to walk towards O at 3 miles an hour. How far is each man from O after walking for x hours?

The men are then 5 miles apart from each other; form an equation to find x and solve it.

11. A quadrilateral $ABCD$, which has its sides AB and AD equal, can be inscribed in a circle. The diagonals AC and BD intersect at X. Prove that

   (i) the angles ACB and ACD are equal,

   (ii) the triangles ABC and DXC are similar,

   (iii) $CX.CA = CB.CD$.

12. The diagram represents a side view of a cubical box with the lid partly open. The length of an edge OZ of the box and of the lid OC is 10 in. and the lid is raised through an angle of 30°. **Calculate** the distances of C from X and Z, giving your answer to three significant figures.