Oxford and Cambridge Schools Examination Board.

School Certificate Examination.

ELEMENTARY MATHEMATICS II.

FRIDAY, JULY 15TH, 1949. 2 HOURS.

[*Answer all the questions in Section* A (1–4) *and not more than* **five** *of the questions in Section* B (5–12).

Logarithms, slide-rules, or algebra may be used in any question, unless directions to the contrary are given.]

SECTION A.

1. Find the yearly interest on £2540 at $1\frac{3}{4}$ per cent. per annum.

Also find what sum of money will produce a yearly interest of £26. 13*s.* at $2\frac{1}{2}$ per cent. per annum.

[Neither logarithms nor slide-rules may be used in answering this question.]

2. Show that $2x-3$ is a factor of $2x^3-3x^2-18x+27$, and find the other two factors.

Hence solve the equation

$$2x^3-3x^2-18x+27=0.$$

3. $ABCD$ is a square. L is the middle point of the side BC and M is the middle point of the side CD. Prove that the triangles ABL and BCM are congruent.

The lines AL and BM intersect at X. Prove that the angle AXM is a right angle.

4. In a rhombus $ABCD$ the lengths of the diagonals AC and BD are respectively 10 cm. and 12 cm. **Calculate** the angles DAB and ABC to the nearest minute.

[You may assume that the diagonals of a rhombus are at right angles to each other.]

Also **calculate** the length of the side AB to the nearest hundredth of a centimetre.

Section B.

5. Draw the graphs of

$$y=x^2 \text{ and } y=\tfrac{1}{2}(x+4)$$

for values of x from -2 to 2, using the same axes for both graphs; take 1 in. (or 2 cm. if you use centimetre paper) to represent the unit.

Hence find the roots of the quadratic equation

$$2x^2=x+4$$

as accurately as you can.

6. Draw a line 3 in. long and on this line as diagonal **construct** a square, using ruler and compasses, but not set-square or protractor.

Construct a line parallel to a side of the square so as to divide the square into two rectangles of which the areas are in the ratio 2 : 3.

[No proofs or explanations are needed, but all construction lines must be clearly shown.]

7. A rectangular airfield is represented on a map, of which the scale is 1 centimetre to 1 kilometre, by a rectangle measuring 3·5 cm. by 2 cm. Show that the area of the airfield is approximately 1730 acres.

[1 acre = 4840 sq. yd., 1 metre = 39·37 in.]

8. It is given that

$$x=1+\frac{p}{p-q} \quad \text{and} \quad y=2-\frac{3q}{p+q}.$$

Prove that

(i) $\dfrac{x}{y}=\dfrac{p+q}{p-q}$;

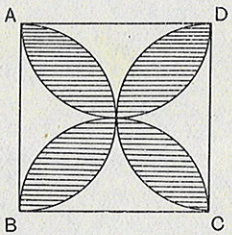
(ii) $\dfrac{3}{x}+\dfrac{1}{y}=2$.

9. In a parallelogram $ABCD$, M is the middle point of the diagonal AC. A line through M (not parallel to AB or AD) cuts AB at P and DC at Q. Prove that

(i) AP is equal to CQ,

(ii) BQ is equal to DP.

10. The shaded portion of the diagram represents a decoration which is cut out of thin metal. This decoration is constructed by taking a square $ABCD$ of side 4 in. and describing a semi-circle on each side. By subtracting the area of a triangle from that of a semi-circle, or otherwise, prove that the area of the decoration is $(8\pi - 16)$ sq. in.



11. The numbers a, b and c are chosen so that

$$2x^2 - 3x - 7 = a(x-1)(x-2) + b(x-2)(x-3) + c(x-3)(x-1)$$

for all values of x. Find the value of a by substituting 3 for x in both sides of the equation.

Also find the values of b and c.

Hence, or otherwise, show that

$$\frac{2x^2 - 3x - 7}{(x-1)(x-2)(x-3)} = \frac{1}{x-3} - \frac{4}{x-1} + \frac{5}{x-2}.$$

12. The length of the edge of a cube is 2 in. $ABCD$ is one of the faces of the cube and AP is the third edge through A. M is the middle point of AP. Make an accurate drawing of the triangle MBD.