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Lower I

1950
SCOTTISH LEAVING CERTIFICATE EXAMINATION

MATHEMATICS

LOWER GRADE—(FIRST PAPER)

Monday, 13th March—9.30 A.M. to 11.30 A.M.

Before attempting to answer any question, candidates should read the whole of it very carefully, since time is often lost through misapprehension as to what is really required.

Four-place logarithmic tables are provided.

All the figures should be neatly drawn, and, where it is necessary to turn over a page during the answer to a question, a rough copy of the figure MUST *be drawn on the fresh page. All the steps of the proofs must be given. Preference will be given to proofs which depend on first principles, and in all cases it should be clearly shown on what assumptions the demonstrations are based. Where geometrical references are necessary in written proofs, care should be taken to ensure that such references are clear and intelligible. Text-book reference numbers, apart from those of Euclid, should* NOT *be used.*

The value attached to each question or part of a question is shown in the margin.

Marks will be deducted for careless or badly arranged work.

SECTION I

All the questions in this Section should be attempted.

Marks

1. Prove that the medians of a triangle are concurrent and that the point of concurrency is a point of trisection of each median. 11

2. Prove that an angle at the centre of a circle is twice any angle at the circumference which stands on the same arc. 12

3. Prove that, if M is the mid-point of the side BC of any triangle ABC, then $AB^2 + AC^2 = 2\,AM^2 + 2\,BM^2$. 11

4. Prove that, if two triangles have the sides of the one proportional to the sides of the other, the triangles are equiangular, those angles being equal which are opposite corresponding sides. 12

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SECTION II

Only THREE *questions should be attempted from this Section.*

The propositions in Section I (above) on which certain of these deductions depend are indicated in brackets.

Marks

5. Two circles, with centres B and C, are such that the circumference of each passes through the centre of the other. CB produced meets the first circle at D. The circles intersect at P and Q. 2

Prove that—

(i) angle $PBQ = 120°$; 5

(ii) the triangle PDQ is equilateral; 6

(iii) PQ is a tangent to the circle DBQ. 5

(Section I, 2.)

6. ABC is a triangle in which $AB = 8$ cm., $BC = 24$ cm. and $CA = 25$ cm.

(i) State whether the triangle ABC is acute, obtuse or right-angled, giving your reasons. 5

(ii) Calculate the length of the projection of AB on AC. 6

(iii) The parallelogram $ABCD$ is completed. Calculate the length of the diagonal BD. 7

(Section I, 3.)

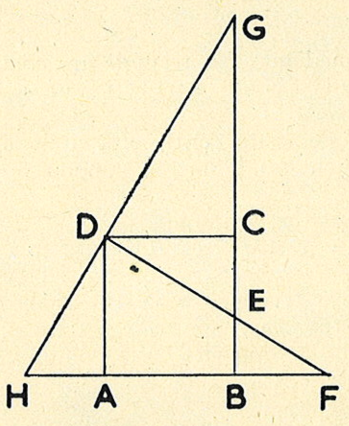
7. In the accompanying figure, which need not be copied into your examination book, $ABCD$ is a square. DEF is a straight line cutting BC at E and AB produced at F. GDH is a straight line at right angles to DEF and cutting BC produced at G and BA produced at H.

Prove that—

(i) $DG = DF$; 6

(ii) rect. $EC . CG$ = square $ABCD$; 6

(iii) $EG = CD(\tan\alpha + \cot\alpha)$, where α denotes angle BHG. 6



Marks

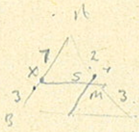
8. In a triangle ABC a point X on AB is taken such that $\frac{AX}{XB} = \frac{7}{3}$ and a point Y on BC such that $\frac{BY}{YC} = \frac{2}{3}$. XM is drawn parallel to BC cutting AC in M and YN is drawn parallel to BA cutting AC in N. XM and YN intersect at S. 2

If AC measures 20 inches, calculate—

(i) the length of AM ; 5

(ii) the length of NM ; 6

(iii) the ratio $SM : YC$. 5



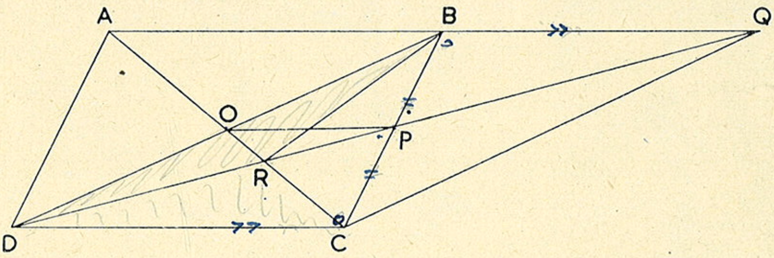
9. In the figure, which need not be copied into your examination book, $ABCD$ is a parallelogram with O the intersection of the diagonals and P the mid-point of BC. DP cuts AC at R and is produced to meet AB produced at Q. BR, OP and QC are drawn.

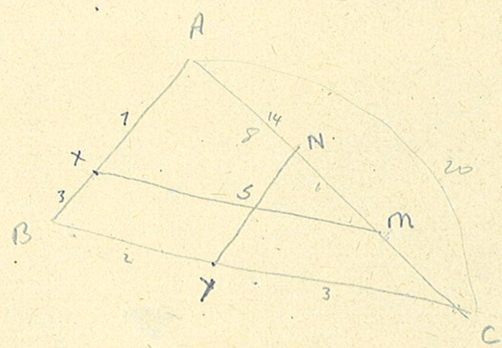
Prove that—

(i) $DCQB$ is a parallelogram ; 6

(ii) triangle DRB = triangle DRC in area ; 6

(iii) $OP = \frac{1}{4} AQ$. 6





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